Bid Coordination in Split-award Procurement: The Buyer Need not Know Anything

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Abstract

Anton and Yao (1989) show that in split-award procurement auctions bidders coordinate their bids to sustain high buyer price. We relax their assumption that the buyer has full information about the suppliers' production costs and restore the coordination outcome.

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1 Introduction

Anton and Yao (1989), hereafter AY, established the surprising result that under complete information about each other's costs if two bidders were allowed to bid for a continuum of splits of a given production requirement of a buyer, the bidders would coordinate their bids leading to a high price for the buyer.[1] Since for any split $\alpha \in (0,1)$, with α fraction of production awarded to a developer D and $1-\alpha$ fraction awarded to a second source S, either bidder can veto the split by submitting a high own bid; the main discipline on the equilibrium price and the viability of an interior split comes from the bidders' sole-source bids. At an interior split a high overall price and individual bidder profits are maintained using sole-source profits as thresholds and threat points.

One notable aspect of AY's setup is the assumption that the buyer has full information about the supplers' costs. This assumption plays an important role when a tie in minimal total bids occurs: among the tied splits the buyer should select one that involves the minimal production cost. But with such knowledge there is no reason for the buyer to hold an auction. Instead, he can make a take-it-or-leave-it joint offer of a price equal to the minimum total production cost which the suppliers cannot refuse, thus avoiding the coordination outcome. Furthermore, in practice, it is very unlikely for a buyer to be fully aware of the suppliers' costs.

In this Note, we assume instead that the buyer has no information about the suppliers' costs. To accommodate this assumption, we use an intuitively plausible tie-breaking rule that works independently of the buyer's information. This tie-breaker first looks at all splits associated with the minimum total bid, then picks the split that is closest to the equal-share split. If this process results in two different splits equidistant from $\alpha = 1/2$, then the tie-breaker can favor either bidder, say bidder D, giving him the option to choose between the two splits and if he does not exercise his option then bidder S selects her desired split. We re-establish the bid coordination outcome under this weaker assumption.

2 Two bidders game

Two potential suppliers, D (developer) and S (second source), submit sealed bids for a continuum of splits, $\alpha \in [0,1]$, of a total production contract x. A pair of bids $(P_D(\alpha), P_S(\alpha))$ implies that at split α, D produces α share for a payment of $P_D(\alpha)$ while S produces $1-\alpha$ share for $P_S(\alpha)$, leading to a buyer price $P_D(\alpha) + P_S(\alpha)$. The bid functions are not required to be smooth. Let $(C_D(\alpha), C_S(\alpha))$ be the respective production costs of D and S at split α with $C_D(0) = C_S(1) = 0$. As in AY, there are no additional restrictions on the cost functions, and the suppliers are assumed to be fully informed about each other's costs when they bid. Total production cost at split α is

$$B(\alpha) = C_D(\alpha) + C_S(\alpha).$$

[1] There are significant follow-on works based on Anton and Yao (1989). See, for example, Alcalde and Dahm (2013) and the references therein.

The bidders' profits are given by

$$\Pi_i(\alpha) = P_i(\alpha) - C_i(\alpha), \quad i = D, S.$$

■ *The buyer's selection of the production split.* For any pair of bids, the buyer chooses a split α to minimize its procurement cost:

$$\min_{\alpha \in [0,1]} \quad G(\alpha) := P_D(\alpha) + P_S(\alpha).$$

If the solution is unique, the buyer chooses this production split. If the minimization yields more than one solution, a tie-breaking rule is needed to pick one split. AY assume that the buyer knows the cost-minimizing split and chooses that split when a tie occurs. As mentioned earlier, if a buyer has such information about the costs, there is no point in holding an auction – it can simply make a take-it-or-leave-it offer, minimizing its procurement cost and avoiding the coordination problem.

We assume, instead, that the buyer has no information about the suppliers's production costs. To accommodate this new assumption, we propose the following tie-breaking rule:

First determine α value(s) closest to $1/2$.

1. If this α value is unique, choose the corresponding production split.

2. If there are two α values equidistant from $1/2$, let bidder D get the priority to declare his preference ordering over these two splits.

If D declares a strict preference for one α over another, pick D's preferred α as the final split. If D expresses an indifference, then S gets to pick her preferred α from the two values which then becomes the final split. If S is also indifferent then the buyer selects the higher of the two α's. $\quad ||$

■ *Equilibrium analysis*

Lemma 1 (AY, 1989) *Let* (P_D^*, P_S^*) *be a Nash equilibrium and* g^* *be the corresponding price to the buyer. Then,*

$$g^* = P_D^*(1) = P_S^*(0).$$

Lemma 1 defines the ceiling on the equilibrium price through sole-source bids. We omit the proof because it is the same as in AY.

Lemma 2 (Production Costs) *Suppose an inefficient split,* $\alpha^{\text{in}} \in [0, 1]$*, is supported in an equilibrium* (P_D^*, P_S^*)*. Then,*

$$\min\{B(0), B(1)\} \geq B(\alpha^{in}). \tag{1}$$

Proof. By Lemma 1,

$$g^* = P_D^*(1) = P_S^*(0) = P_D^*(\alpha^{\text{in}}) + P_S^*(\alpha^{\text{in}}).$$

Without loss of generality, suppose $B(1) \leq B(0)$. Suppose contrary to (1), $C_D(1) < C_D(\alpha^{\text{in}}) + C_S(\alpha^{\text{in}})$. Then

$$0 \leq \Pi_D^*(\alpha^{\text{in}}) \leq g^* - [C_D(\alpha^{\text{in}}) + C_S(\alpha^{\text{in}})] < g^* - C_D(1),$$

where $\Pi_D^*(\alpha^{\text{in}})$ is D's profit in the posited equilibrium involving α^{in}-split. But then D can lower his bid slightly below g^* at the sole-source and realize a profit arbitrarily close to $g^* - C_D(1)$ that exceeds $\Pi_D^*(\alpha^{\text{in}})$, contradicting that α^{in}-split is an equilibrium outcome. Hence, (1) must hold. **Q.E.D.**

Lemma 2 implies that no strictly inefficient split can be supported in a Nash equilibrium if sole-source production is cost efficient.

Proposition 1 (Equilibrium characterization) *Bidding strategies* (P_D^*, P_S^*) *constitute a Nash equilibrium resulting in an equilibrium split* α^* *if and only if the following complete set of conditions under **[1]**-**[3]** are satisfied:*

1. Price ceiling condition:

$$g^* = P_D^*(1) = P_S^*(0). \tag{2}$$

2. No profitable deviation in bidding: *Neither bidder finds it profitable to deviate unilaterally to an alternative bidding strategy, i.e.,*

$$\Pi_i^*(\alpha^*) + B(\alpha^*) \leq \Pi_i^*(\alpha) + B(\alpha) \quad \textit{for all } \alpha \in [0,1], \quad i = D, S. \tag{3}$$

3. Picking the winning split α^* using the buyer's selection rule and the tie-breaker, given submitted bids (P_D^*, P_S^*):

(i) If $|\alpha - \frac{1}{2}| < |\alpha^* - \frac{1}{2}|$, *then*

$$g^* < P_D^*(\alpha) + P_S^*(\alpha); \tag{4}$$

(ii) If $|\alpha^* - \frac{1}{2}| < |\alpha - \frac{1}{2}|$, *then*

$$g^* \leq P_D^*(\alpha) + P_S^*(\alpha); \tag{5}$$

(iii) If $|\alpha^* - \frac{1}{2}| = |\alpha - \frac{1}{2}|$, *then*

– *either (a):*

$$g^* < P_D^*(\alpha) + P_S^*(\alpha), \tag{6}$$

– *or (b):*

$$g^* = P^*_D(\alpha) + P^*_S(\alpha), \quad \textit{and} \tag{7}$$

$$\begin{cases} \Pi^*_D(\alpha^*) > \Pi^*_D(\alpha); \\ \underline{\textit{or}} \quad \Pi^*_D(\alpha^*) = \Pi^*_D(\alpha) \\ \quad \textit{and } \Pi^*_S(\alpha^*) > \Pi^*_S(\alpha); \\ \underline{\textit{or}} \quad \Pi^*_D(\alpha^*) = \Pi^*_D(\alpha) \\ \quad \Pi^*_S(\alpha^*) = \Pi^*_S(\alpha) \\ \quad \textit{and } \alpha^* > \alpha. \end{cases} \tag{8}$$

Proof. [**Necessity**] The necessity of item [**1**] follows from Lemma 1. The derivation of condition (3) in item [**2**] is exactly the same as in AY.

To verify the necessity of item [**3**], first observe that α^* being the winner, it must pick itself when faced with all alternative values $\alpha \neq \alpha^*$. The conditions are exhaustively listed by partitioning the range of production splits, $[0,1]$. In the range under (i), if condition (4) fails for some α then the tie-breaker would discard α^* as the winner, so (4) must hold. For α in the range listed under (ii), even if the overall bid price equals g^* the tie-breaker will pick α^*, implying condition (5). For the unique α under (iii), either the overall price must be higher than g^* implying (6), or in the case of a tie between α^* and α the second tie-breaking provision is implemented implying conditions (7) and (8).

[**Sufficiency**] The proof is straightforward and omitted. **Q.E.D.**

Proposition 2 (Sole-source outcome) **(i)** *If* $B(0) < B(\alpha)$ *for all* $\alpha \in (0,1]$*, the sole-source contract awarded to the cost-efficient supplier* S *is the unique Nash equilibrium outcome with bids*

$$\begin{cases} g^* = B(1) = P^*_D(1) = P^*_S(0), \\ P^*_D(\theta) > g^*, \ \ P^*_S(\theta) > g^*, \ \ \textit{for all } \theta \in (0,1), \end{cases} \tag{9}$$

and yielding profits $\Pi^*_S = B(1) - B(0)$*, and* $\Pi^*_D = 0$*.*

(ii) *Let* $B(0) = B(1)$ *and* $B(0) \leq B(\alpha)$ *for all* $\alpha \in (0,1)$*. Then sole-source contract awarded to bidder* S *or* D *can be supported as an equilibrium outcome.*

Proof. **(i)** Lemma 2 shows that no inefficient split $\alpha \in (0,1]$ can be supported in an equilibrium if sole-source production by S is cost-efficient. The sole-source equilibrium result involving bidder S follows by applying Proposition 1. The only points that need clarifications are the bids at the interior splits $\theta \in (0,1)$ and the use of our tie-breaker. At all interior splits S and D individually submit high enough bids, in excess of g^*, to make split production unattractive for the buyer. (If their bids were to add up to g^*, the tie-breaker would rule out sole-source outcome.) The bid specifications satisfy the sufficient condition (3) characterized in Proposition 1 because:

$$\begin{aligned}\Pi_S^*(0) + B(0) = B(1) - C_S(0) + B(0) = B(1) \quad &< \quad g^* - C_S(\theta) + C_D(\theta) + C_S(\theta) \\ &< \quad P_S^*(\theta) - C_S(\theta) + C_D(\theta) + C_S(\theta) \\ &= \quad \Pi_S^*(\theta) + B(\theta) \ \text{ for all } \theta \in (0,1).\end{aligned}$$

Given the ties $g^* = P_D^*(1) = P_S^*(0)$, the tie-breaker would first give an option to bidder D to choose between $\alpha = 1$ and $\alpha = 0$. Since D earns zero profit for either choice, he, let's say, declares indifference and then S gets to choose her preferred outcome which is $\alpha = 0$.

(ii) If there are multiple cost-efficient splits, including sole-source production by either supplier and possibly some interior splits, the same strategies (9) will result in sole-source award to D or S as an equilibrium outcome. The same proof as in part **(i)** applies except that now D is indifferent between $\alpha = 1$ and $\alpha = 0$ and he can induce either outcome by expressing a strict preference for it in the tie-breaker. **Q.E.D.**

In part **(i)** of Proposition 2 we deliberately chose S to be more cost-efficient than D in order to illustrate the full application of our tie-breaking rule. The case that D is the most cost-efficient supplier works in an analogous way. Part **(ii)** of Proposition 2 shows that either sole-source outcome is possible when both arrangements are cost-efficient along with possibly other interior splits.

Proposition 3 (Split-award, multiple equilibria, efficient & inefficient outcomes) (i) *Suppose for any $\alpha \in (0,1)$ the inequality (1) fails, i.e.,* $\min\{B(1), B(0)\} < B(\alpha)$. *Then the interior split, α, cannot be supported in equilibrium.*

(ii) *Let $B(1) \leq B(0)$ and $N = \{\alpha | B(\alpha) \leq B(1),\ 0 < \alpha < 1\}$ be the set of outcomes for which joint production costs are less than sole-source production costs. Then, any $\alpha \in N$ can be supported as an equilibrium outcome.*

Proof. **(i)** This is implied by Lemma 2.

(ii) Below we construct bidding strategies $(P_D^*(\theta), P_S^*(\theta))$, $\theta \in [0,1]$ to support any $\alpha \in N$ as an equilibrium outcome.

For α to be an equilibrium split there must be some g^* such that

$$\begin{cases} P_D^*(1) = P_S^*(0) = g^* = P_D^*(\alpha) + P_S^*(\alpha), \quad \text{(by Lemma 1)} \\ \qquad\qquad \Pi_D^*(\alpha) \geq \Pi_D^*(1), \quad \Pi_S^*(\alpha) \geq \Pi_S^*(0). \end{cases} \tag{10}$$

Let

$$P_D^*(\theta) = P_S^*(\theta) = g^* + \varepsilon, \quad \text{where } \ \varepsilon > 0, \ \text{ for all } \ \theta \in (0,1) \setminus \{\alpha\}. \tag{11}$$

Given conditions (10) and (11), the candidate splits for equilibrium are $\theta = 1, 0, \alpha$. With total bids tied at $\theta = 1, 0, \alpha$, our tie-breaker will select uniquely $\theta = \alpha$.

Now determine g^* by setting[2]

$$\begin{aligned} g^* - B(\alpha) &= g^* - B(1) + g^* - B(0), \\ \text{i.e.,} \quad g^* &= B(1) + B(0) - B(\alpha) > 0. \end{aligned} \tag{12}$$

It remains to verify the profit (weak) inequality conditions in (10) and explicitly derive $P_D^*(\alpha)$ and $P_S^*(\alpha)$. Let

$$\begin{aligned} \Pi_D^*(\alpha) = P_D^*(\alpha) - C_D(\alpha) &= g^* - B(1) = \Pi_D^*(1), \quad \text{(verifies profit inequality condition for D)} \\ \text{so that } P_D^*(\alpha) &= g^* - B(1) + C_D(\alpha), \qquad (13) \\ \text{that, in turn, implies } P_S^*(\alpha) &= g^* - P_D^*(\alpha) \\ &= B(1) - C_D(\alpha). \quad \text{(using (10))} \qquad (14) \end{aligned}$$

Verify the profit inequality condition for S as follows:

$$\begin{aligned} \Pi_S^*(\alpha) &= P_S^*(\alpha) - C_S(\alpha) \\ &= B(1) - C_D(\alpha) - C_S(\alpha) \\ &= g^* - B(0) = \Pi_S^*(0). \end{aligned}$$

This completes the equilibrium argument, by construction, for the split $\alpha \in N$.

Finally, note that no other $\theta \in (0, 1)$ can be supported in equilibrium for the proposed strategies $(P_D^*(\theta), P_S^*(\theta))$ defined by (10), (11), (13) and (14), establishing the equilibrium outcome α. **Q.E.D.**

Part **(ii)** of Proposition 3 admits a case common with part **(ii)** of Proposition 2: $B(1) = B(0) = B(\alpha)$ for some $0 < \alpha < 1$. In such situations either sole-source or split-award can arise in equilibrium.

Another central message of Proposition 3 is that when sole-source production does not strictly lower costs relative to split production, potentially efficient and many inefficient split-award outcomes can be supported in equilibrium. This multiplicity is due to both the extensive strategic flexibility each bidder enjoys in vetoing any interior split as well as each bidder's control over the other bidder's strategic maneuver through sole-source bids. However, as we will see next the bidders are able to focus on a cost-efficient equilibrium that Pareto-dominates all inefficient equilibria.

Proposition 4 (Pareto-dominant efficient equilibrium) *Any cost-efficient split $\alpha^{eff} \in [0, 1]$ strictly Pareto-dominates all equilibria involving inefficient splits from the bidders' perspective. This domination is achieved by a single pair of bidding strategies supporting α^{eff} in a Nash equilibrium with buyer price at its maximal value.*

Proof. If $\alpha^{\text{eff}} \in \{0, 1\}$, no inefficient split can be supported as an equilibrium outcome (by Lemma 2). Fix an efficient split $\alpha^{\text{eff}} \in (0, 1)$ and choose the specific equilibrium constructed in the proof of

[2] It can be easily shown that $B(1) + B(0) - B(\alpha)$ is the maximal possible buyer price in any equilibrium involving α-split.

Proposition 3 to support α^{eff} so that

$$g^{\text{eff}} = B(1) + B(0) - B(\alpha^{\text{eff}}). \tag{15}$$

Consider any inefficient split α^{in}. The maximal buyer price supporting α^{in} is:

$$g^{\max} = B(1) + B(0) - B(\alpha^{\text{in}}).$$

Take any equilibrium supporting α^{in}, with buyer price no higher than $g^{\max}$. In this equilibrium,

$$\Pi_D(\alpha^{\text{in}}) \geq g^{\max} - B(1); \qquad \Pi_S(\alpha^{\text{in}}) \geq g^{\max} - B(0). \tag{16}$$

Also,

$$\Pi_D(\alpha^{\text{in}}) + \Pi_S(\alpha^{\text{in}}) \leq g^{\max} - B(\alpha^{\text{in}}). \tag{17}$$

We claim

$$\Pi_D^*(\alpha^{\text{eff}}) > \Pi_D(\alpha^{\text{in}}) \tag{18}$$

$$\text{and} \quad \Pi_S^*(\alpha^{\text{eff}}) > \Pi_S(\alpha^{\text{in}}), \tag{19}$$

which means strict Pareto domination.

Suppose, contrary to our claims, (18) is false[3] so that

$$\Pi_D(\alpha^{\text{in}}) \geq \Pi_D(\alpha^{\text{eff}}) = g^{\text{eff}} - B(1). \tag{20}$$

(The second equality above follows by construction of the efficient equilibrium in the same way $\Pi_D^*(\alpha) = g^* - B(1)$ in the proof of Proposition 3.)

Now we can write

$$\begin{aligned}
g^{\max} - B(0) &\underbrace{\leq}_{\text{(by (16))}} \Pi_S(\alpha^{\text{in}}) \underbrace{\leq}_{\text{(by (17))}} g^{\max} - B(\alpha^{\text{in}}) - \Pi_D(\alpha^{\text{in}}) \\
&\qquad\qquad\qquad \underbrace{\leq}_{\text{(using (20))}} g^{\max} - B(\alpha^{\text{in}}) - g^{\text{eff}} + B(1)
\end{aligned}$$

$$\begin{aligned}
\text{i.e.,} \quad & -B(0) \leq -B(\alpha^{\text{in}}) - g^{\text{eff}} + B(1) \\
\text{i.e.,} \quad & B(0) + B(1) - B(\alpha^{\text{eff}}) - B(0) - B(1) \leq -B(\alpha^{\text{in}}) \qquad \text{(using (15))} \\
\text{i.e.,} \quad & B(\alpha^{\text{eff}}) \geq B(\alpha^{\text{in}}),
\end{aligned}$$

which is a contradiction.

Hence (18) must hold. By a similar logic, (19) must also hold. This completes the proof. **Q.E.D.**

This Pareto-domination result is achieved by a single efficient equilibrium constructed explicitly

[3] We are not taking any position yet with regard to (19).

that dominates over *all* inefficient equilibria. Proposition 4 summarises the central message of this note: the bidders coordinate implicitly on a cost-efficient Pareto-dominating equilibrium. In this equilibrium, the bidders' joint profits and individual profits are maximal. But it leads to the highest maximal procurement price for the buyer. The buyer does not need to know anything about the suppliers' costs for bid coordination to occur.

3 Remarks

When the buyer does not have any information regarding suppliers' costs, we suggest the following tie-breaker to deal with three bidders:

– If the minimum total bid from submitted bids result in ties of production shares $\underset{\sim}{\alpha}_j = (\alpha_{j,1}, \alpha_{j,2}, \alpha_{j,3})$ where $\sum_{i=1}^{3} \alpha_{j,i} = 1$ and T is the set of ties containing $\underset{\sim}{\alpha}_j$, let $\underset{\sim}{\alpha} = (\alpha_1, \alpha_2, \alpha_3) = \arg\min_{\underset{\sim}{\alpha}_j \in T} \sum_{i=1}^{3} |\alpha_{j,i} - \frac{1}{3}|$. If $\underset{\sim}{\alpha}$ is unique, choose this split for the contract award.

– If there is more than one such $\underset{\sim}{\alpha}$, give selection priorities in the following order, bidder 1 $\succ$ bidder 2 $\succ$ bidder 3, with indifference expressed by the previous bidder passing the option of choice onto the next bidder. If all three bidders express indifference, implement the allocation with the highest $\alpha_{j,1}$, and if all $\alpha_{j,1}$ are equal then choose one with the highest $\alpha_{j,2}$; this last step would determine $\alpha_{j,3}$ uniquely given that production shares must add up to 1. ||

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